

Math Camp

Day 1 Exercises

Solutions

Exercise 1

$$\text{a) } f'(x) = \frac{9}{\sqrt{x}}$$

$$\text{b) } f'(t) = 54t^8 + 75t^4 - 64t^3$$

$$\text{c) } f'(x) = 35x^6 - 18x^5$$

$$\text{d) } f'(y) = 72y^2(6y^3 + 9)^3$$

$$\text{e) } f'(x) = -\frac{21x^2+13}{(7x^3+13x+3)^2}$$

$$\text{f) } f'(u) = \frac{42u+56}{(2u+5)^3}$$

Exercise 2

a) increasing, convex

b) increasing, convex

c) stationary, convex

Exercise 3

$$\text{a) } \frac{\partial f}{\partial x_1} = 3x_1^2 - 9x_2$$

$$\frac{\partial^2 f}{\partial x_1^2} = 6x_1$$

$$\frac{\partial f}{\partial x_2} = -6x_2 - 9x_1$$

$$\frac{\partial^2 f}{\partial x_2^2} = -6$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = \frac{\partial^2 f}{\partial x_2 \partial x_1} = -9$$

$$\text{b) } \frac{\partial f}{\partial x_1} = 21(7x_1 + 3x_2)^2$$

$$\frac{\partial^2 f}{\partial x_1^2} = 2058x_1 + 882x_2$$

$$\frac{\partial f}{\partial x_2} = 9(7x_1 + 3x_2)^2; \frac{\partial^2 f}{\partial x_2^2} = 378x_1 + 162x_2; \frac{\partial^2 f}{\partial x_1 \partial x_2} = \frac{\partial^2 f}{\partial x_2 \partial x_1} = 882x_1 + 378x_2$$

Exercise 4

$$\text{a) } f'(x) = \frac{3}{x}$$

$$\text{b) } f'(x) = \frac{2 \ln 8x}{x}$$

$$\text{c) } f'(x) = \ln \frac{(2x+5)}{x(2x+5)}$$

Exercise 5

Differentiate each of the following exponential function.

$$\text{a) } f'(x) = 2e^{2x}$$

$$\text{b) } f'(x) = xe^{5x}(5x + 2)$$

$$\text{c) } f'(x) = \frac{10e^{5x}}{(e^{5x}+1)^2}$$

Exercise 6

Evaluate the following definite integrals.

$$\text{a) } \int_0^6 5x dx = 90$$

$$\text{b) } \int_0^{10} 2e^{-2x} dx = 1 - \frac{1}{e^{20}} \approx 1$$

$$\text{c) } \int_0^3 8x(2x^2 + 3) dx = 432$$