

# Math Camp

## Day 2 Exercises

### Exercise 1

Find the extrema for the following functions by:

- 1) finding the critical value(s), and
- 2) determining if at the critical value(s) the function is at a maximum, a minimum, or an inflection point.

a)  $f(x) = 3x^3 - 36x^2 + 135x - 13$

b)  $f(x) = 2x^4 - 16x^3 + 32x^2 + 5$

c)  $f(x) = -(x - 8)^4$

c)  $f(x) = (5 - x)^3$

### Exercise 2:

For each of the following functions:

- 1) find the critical points at which the function may be optimized, and
- 2) determine whether at these points the function is maximized, minimized, is at an inflection point, or is at a saddle point.

a)  $f(x_1, x_2) = 3x_1^2 - x_1x_2 + 2x_2^2 - 4x_1 - 7x_2 + 12$

b)  $f(x_1, x_2) = 60x_1 + 34x_2 - 4x_1x_2 - 6x_1^2 - 3x_2^2 + 5$

### Exercise 3:

a) Use the Lagrange multiplier method to solve the following problem:

$$\text{Max } f(x_1, x_2) = 4x_1^2 - 2x_1x_2 + 6x_2^2 \quad \text{subject to} \quad x_1 + x_2 = 72$$

b) Estimate the effect on the value of the objective function from a 1-unit increase in the constant of the constraint.

c) Check the accuracy of your estimation by optimizing the original objective function to a new constraint:  $x_1 + x_2 = 73$