Math Camp

Day 1 Exercises

Solutions

Exercise 1

a) $f'(x) = \frac{9}{\sqrt{x}}$ b) $f'(t) = 54t^8 + 75t^4 - 64t^3$ c) $f'(x) = 35x^6 - 18x^5$

d) $f'(y) = 72y^2(6y^3 + 9)^3$ e) $f'^{(x)} = -\frac{21x^2+13}{(7x^3+13x+3)^2}$ f) $f'(u) = \frac{42u+56}{(2u+5)^3}$

Exercise 2

a) increasing, convex b) increasing, convex c) stationary, convex

Exercise 3

a) $\frac{\partial f}{\partial x_1} = 3x_1^2 - 9x_2$ $\frac{\partial^2 f}{\partial x_1^2} = 6x_1$

$\frac{\partial f}{\partial x_2} = -6x_2 - 9x_1$ $\frac{\partial^2 f}{\partial x_2^2} = -6$ $\frac{\partial^2 f}{\partial x_1 \partial x_2} = \frac{\partial^2 f}{\partial x_2 \partial x_1} = -9$

b) $\frac{\partial f}{\partial x_1} = 21(7x_1 + 3x_2)^2$ $\frac{\partial^2 f}{\partial x_1^2} = 2058x_1 + 882x_2$

$\frac{\partial f}{\partial x_2} = 9(7x_1 + 3x_2)^2$; $\frac{\partial^2 f}{\partial x_2^2} = 378x_1 + 162x_2$; $\frac{\partial^2 f}{\partial x_1 \partial x_2} = \frac{\partial^2 f}{\partial x_2 \partial x_1} = 882x_1 + 378x_2$

Exercise 4

a) $f'(x) = \frac{3}{x}$ b) $f'(x) = \frac{2ln8x}{x}$ c) $f'(x) = \frac{(2x+5)}{x(2x+5)}$

Exercise 5

Differentiate each of the following exponential function.

a) $f'(x) = 2e^{2x}$ b) $f'(x) = xe^{5x}(5x + 2)$ c) $f'(x) = \frac{10e^{5x}}{(e^{5x}+1)^2}$

Exercise 6

Evaluate the following definite integrals.

a) $\int_0^6 5x dx = 90$ b) $\int_0^{10} 2e^{-2x} dx = 1 - \frac{1}{e^{20}} \approx 1$ c) $\int_0^3 8x(2x^2 + 3) dx = 432$