Math Camp

Day 3 Exercises

Solutions

Exercise 1

a) and b):

Y (number of heads)	$Y = 0$	$Y = 1$	$Y = 2$
Probability density	0.25	0.50	0.25
Cumulative density	0.25	0.75	1.00

c) $\mu_Y = E(Y) = 1$ and $Var(Y) = E(Y^2) - \mu_Y^2 = 0.5$

Exercise 2

$a)\ E(Y) = 0.95$

b) P$(Y = 0) = 1 - P(Y = 1) = 0.05 = 1 - E(Y)$

c) $E(Y|X = 0) = 0.9403$ and $E(Y|X = 1) = 0.9797$

d) $1 - E(Y|X = 1) = 0.0203$ and $1 - E(Y|X = 0) = 0.0597$

e) $P(X = 1|Y = 0) = 0.1$ and $P(X = 0|Y = 0) = 0.9$

f) No

Tip: Requires application of joint distribution measures (conditional probabilities and conditional expectations).

Exercise 3

a) $\mu_Y = \$1{,}000$ and $\sigma_y = \sqrt{\sigma_Y^2} = \sqrt{\$19{,}000{,}000} \approx \$4358.9$

b) $1) E(Y) = \mu_Y = \$1{,}000$ 2) $P(\bar{Y} > 2000) = 1 - P(\bar{Y} \leq 2000) \approx 0.0110$

Tip: Solving (b (2)) requires the application of the central limit theorem stating that $\bar{Y} \sim N\left(\mu_Y, \frac{\sigma_Y^2}{n}\right)$. From there you can again apply z-transformation to obtain the probability that $P(\bar{Y} \leq 2000)$.