Math Camp

Day 3 Exercises

Exercise 1

Let Y denote the number of "heads" that occur when two coins are tossed.

       a) Calculate the probability density of Y.
       b) Calculate the cumulative density of Y.
       c) Calculate the mean and the variance of Y.

Exercise 2:

The following table gives the joint probability distribution between employment status and college graduation among those either employed or looking for work (unemployed) in the working age US population, based on the 1990 US Census.

	Unemployed $(Y = 0)$	Employed $(Y = 1)$	Total
Non-college grads $(X = 0)$	0.045	0.709	0.754
College grads $(X = 1)$	0.005	0.241	0.246
Total	0.05	0.95	1.00

a) Compute $E(Y)$.

b) The unemployment rate is the fraction of the labor force that is unemployed. Show that unemployment rate is given by $(1 - E(Y))$.

c) Calculate $E(Y|X = 1)$ and $E(Y|X = 0)$.

d) Calculate the unemployment rate for (i) college graduates and (ii) non-college graduates.

e) A randomly selected member of this population reports being unemployed. What is the probability that this worker is a (i) college graduate? (ii) a non-college graduate?

f) Are educational achievement and employment status independent?

Exercise 3:

In any year, the weather can inflict storm damage to a home. From year to year, the damage is random. Let Y denote the dollar value of damage in any given year. Suppose that 95% of the years $Y = \$0$, but in 5% of the years $Y = \$20,000$.

a) What is the mean and the standard deviation of the damage in any year?

b) Consider an "insurance pool" of 100 people whose homes are sufficiently dispersed so that in any year, the damage to different homes can be viewed as independently distributed random variables. Let $\bar{Y}$ denote the average damage to these 100 homes in a year.
        1) What is the expected value of the average damage $\bar{Y}$?
        2) What is the probability that $\bar{Y}$ exceeds $2,000?

TABLE 1   CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION (*Continued*)

Cumulative probability

Entries in the table give the area under the curve to the left of the z value. For example, for z = 1.25, the cumulative probability is .8944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990