

# Math Camp

## Day 4 Exercises

### Solutions

#### Exercise 1

a) Solution:  $E(\tilde{Y}) = E\left(\frac{1}{2}Y_1\right) + E\left(\frac{1}{3}Y_2\right) + E\left(\frac{1}{6}Y_3\right) = \frac{1}{2}E(Y_1) + \frac{1}{3}E(Y_2) + \frac{1}{6}E(Y_3) = \frac{1}{2}\mu + \frac{1}{3}\mu + \frac{1}{6}\mu = \mu$

b) Solution:  $var(\tilde{Y}) = var\left(\frac{1}{2}Y_1\right) + var\left(\frac{1}{3}Y_2\right) + var\left(\frac{1}{6}Y_3\right) = \frac{1}{4}var(Y_1) + \frac{1}{9}var(Y_2) + \frac{1}{36}var(Y_3) = \frac{1}{4}\sigma^2 + \frac{1}{9}\sigma^2 + \frac{1}{36}\sigma^2 = \frac{7}{18}\sigma^2 > var(\bar{Y}) = \frac{\sigma^2}{n} = \frac{\sigma^2}{3} = \frac{6}{18}\sigma^2$

c) No, it is less efficient.

#### Exercise 2

a)  $\hat{p} = E(Y) = \frac{215}{400} = 0.5375$

b)  $\sigma_{\hat{p}} = \sqrt{\sigma_p^2} \approx 0.0249$

c) p-value =  $2\Phi(-|t|) \approx 2\Phi(-1.51) \approx 2 * 0.0655 = 0.131$

d) p-value =  $1 - \Phi(t) \approx 1 - \Phi(1.51) \approx 0.0655$

e) Two sided vs one-sided hypothesis test.

f) No, it did not.

g)  $\hat{p} \pm 1.96SE(\hat{p}) = 0.5375 \pm 1.96 * 0.0249 = (0.4887, 0.5863)$

h) Wider. By increase to 99% confidence interval, lower the risk of Type 1 errors (smaller rejection area).

i) Using result from part g) or the result from part c) we cannot reject the Null.