Math Camp

Day 4 Exercises

Exercise 1

Suppose that Y_1, Y_2, Y_3 is a random sample from a $N \sim (\mu, \sigma^2)$ population. To estimate μ, consider the following weighted estimator:

$$\tilde{Y} = \frac{1}{2} Y_1 + \frac{1}{3} Y_2 + \frac{1}{6} Y_3$$

a) Show that $\tilde{Y}$ is an unbiased estimator.

b) Find the variance of $\tilde{Y}$ and compare it to the variance of the sample mean $\bar{Y}$.

c) Is $\tilde{Y}$ as good an estimator as $\bar{Y}$?

Exercise 2:

In a survey of 400 likely voters, 215 responded that they would vote in favor of the initiative for a single, unified health insurance in Switzerland, while 185 responded that they would vote against it. Let p denote the fraction of <u>all</u> likely voters who preferred a unified health insurance at the time of the survey, and let $\hat{p}$ be the fraction of survey respondents who (actually) preferred a unified health insurance.

Notation: Denote each voter's preference by Y. $Y = 1$ if the voter prefers the unified health insurance and $Y = 0$ if the voter does not prefer the unified health insurance. Y is a Bernoulli random variable with probability $P(Y = 1) = p$ and $P(Y = 0) = 1 - p$.

Assume that n is large and use the table included below.

a) Use the survey results to estimate p.

b) Use the estimator of the variance of $\hat{p}$, i.e., $\frac{\hat{p}(1-\hat{p})}{n}$, to calculate the standard error of your estimator.

c) What is the p-value for the test $H_0: p = 0.5$ vs. $H_1: p \neq 0.5$? Assume that n is large and use the table included below.

d) What is the p-value for the test $H_0: p = 0.5$ vs. $H_1: p > 0.5$? Assume that n is large and use the table included below.

e) Why do the results in parts c) and d) differ?

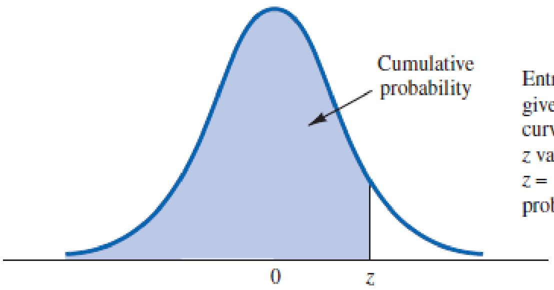
f) Did the survey contain statistically significant evidence that voters were in favor of a unified health insurance at the time of the survey?

g) Construct a 95% confidence interval for p. Assume that n is large and use $z^{critical}$.

h) If you constructed a 99% confidence interval for p and compared it to the one calculated in part g), would it be wider or narrower? Why?

i) Without doing any additional calculations, test the hypothesis $H_0: p = 0.5$ vs. $H_1: p \neq 0.5$ at the 5% significance level.

TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION (*Continued*)



Entries in the table give the area under the curve to the left of the *z* value. For example, for *z* = 1.25, the cumulative probability is .8944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990